Accurate Power Sharing and Synthetic Inertia Control for DC Building Microgrids with Guaranteed Performance

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ABSTRACT Direct current (DC) building microgrids allow the integrations of DC distributed energy resources (DERs) and loads with a simpler topology and eliminate alternating current (AC) system issues such as frequency transients and harmonics. Because of the domination of power-electronics-interfaced generators, islanded DC microgrids in general present very low inertia, which reveals subtle stability threats to the systems. In addition, conventional droop-based DER power sharing mechanisms may suffer from a poor power sharing accuracy and DC bus voltage deviations that require secondary restorations. In this paper, a novel control scheme for building-scale islanded DC microgrids is proposed based on finite control set model predictive control (FCS-MPC). A new DER power sharing mechanism is devised in the FCS-MPC, which eliminates bus voltage deviation during load/generation fluctuation, and offers load/DER plug-and-play capabilities. Moreover, for the first time, virtual capacitance control for DC microgrids is implemented in a model predictive manner to enhance the synthetic inertia of bus voltage. Both simulation and experimental case studies are carried out to provide verification for the promising performance of the proposed method.

INDEX TERMS DC building, DC microgrid, synthetic inertia, virtual capacitance, model predictive control, distributed generation.

I. INTRODUCTION

The worldwide Zero Net Energy (ZNE) building goals imply a higher penetration of renewable DERs and an increasing energy efficiency requirement in both residential and commercial buildings [1]. The configuration of microgrid provides a promising platform to integrate clusters of DERs and loads, which simultaneously maximizes the values of renewables and improves the reliability of utility grid by enabling ancillary service and Virtual Power Plant (VPP) [2], [3]. Because the majority of DERs in building scale microgrids are DC based (e.g., solar PV system, energy storage system (ESS), ultra-capacitor, etc.), and DC loads are becoming increasingly prevalent (e.g., solid-state LED lighting, DC ventilation, DC data center, EV charger, etc.), DC building microgrids have drawn increasing attentions due to their promising merits that AC microgrids cannot offer [4], [5]. For instance, Robert Bosch LLC and NREL have cooperatively installed several DC microgrid pilot demonstrations in the US [6]. With simpler power-electronics interfaces and AC transient stability issues, DC building microgrid allows the ease of centralized control with a lower communication bandwidth [7].
A DC building microgrid (Fig. 1) may consist of multiple DERs on a common bus through exclusive controllable power-electronics interfaces. Critical and normal loads such as DC data center, lighting, and ventilation can plug and play if the bus voltage is stabilized. In islanded mode, which this paper focuses on, DC bus regulation and DER power distributions become primary control objectives as the grid support is unavailable. However, the stochastic nature of renewable DERs and loads, e.g., PV power fluctuations and electric vehicles (EV), have posed the following challenges that may lead to system instability and hinder the full potentials: 1) inaccurate power sharing among DERs, 2) DC bus voltage steady-state deviations due to load or DER fluctuations, and 3) vulnerability of the DC bus voltage due to extremely low system inertia.

In the literature, droop-based methods have been widely employed and extensively explored for power sharing control and bus voltage regulation in DC microgrids. Ref. [9] proposes a $P_{dc} - v_{dc}^2$ droop control to maintain the common DC bus voltage and realize the power sharing among the ESS, while [10] realizes the same goals by superimposing a small AC voltage with frequency droop to the DC bus. Similarly, [11] introduces an improved power-based droop control for DC microgrids, which autonomously converts power flow control to droop control and enables seamless transitions between islanded and grid-connected modes. An observer-based DC voltage droop and current feed-forward control strategy for DC microgrids is presented in [12], which enhances the robustness compared with conventional droop. He et al. in [13] devises a method integrating close-loop current regulation, droop, and coupled virtual impedance coordinated control techniques. There are also other advanced control strategies for DC microgrids based on modified droop control, i.e., the intelligent power sharing scheme based on fuzzy sliding-mode droop [14], adaptive droop with coefficient shifting and circulating current reduction [15], and the nonlinear droop method for better power sharing accuracy and voltage regulation [16]. Although these methods improve the power sharing accuracy and voltage stability to some extent by appropriate modifications, they may suffer from one or more of the following drawbacks: 1) unavoidable DC bus voltage deviation caused by load or generation change due to the droop mechanism, 2) the need of additional voltage restorations from secondary controllers, and 3) the dependence of extra PWM modules to implement the controllers.

There are a number of other methods for voltage regulations in DC microgrids. Kollimalla et al. in [17] utilizes hybrid energy storage system (HES) to accommodate power fluctuations in DC microgrids. This work mainly focuses on the coordination of battery and supercapacitor in compensation for current fluctuations. Ref. [18] presents the modeling, design, and analysis of DC microgrid control and energy management system (EMS) using fuzzy logic, while [19] presents an active control method for DC data center with eight operation modes. Although these methods achieve an optimal operation and emergency support mode, respectively, bus voltage deviation is not addressed in either methods, which may result in nuisance disturbances to sensitive loads.

Moreover, due to the high penetration level of power-electronics-interfaced DERs, microgrids in general manifest low inertia nature, which leads to the vulnerability of bus frequency and voltage in AC and DC networks, respectively. Virtual Synchronous Generator (VSG) technology has been applied to AC microgrid for virtual spinning inertia support using inverter-based DERs. However, due to the decoupling of AC and DC, VSG is not directly applicable for DC microgrids. Note that inertia in DC systems is represented by the stability of bus voltage instead of frequency. To the best of the authors’ knowledge, very few existing schemes have considered inertia enhancement for DC microgrids. Wu et al. in [20] propose a method to increase DC microgrid inertia by the analogy of AC VSG to DC network, where the grid-connected inverter controls the DC bus. This method does enhance the inertia of DC microgrids, however, it is only applicable in grid-connected mode and the voltage deviation is not eliminated.

As an attempt to address the aforementioned limitations, this paper proposes a unified designing approach of FCS-MPC for building-scale DC microgrids featuring accurate power sharing and virtual inertia synthesis. The major contributions can be summarized as follows:

- A novel power sharing mechanism for DERs is proposed in a predictive manner by introducing a current sharing vector to the FCS-MPC cost function;
- Steady-state DC voltage deviation and secondary restoration are eliminated during the fluctuation of loads or generations;
- Virtual capacitance control is integrated in a FCS-MPC approach for the first of its kind, which enables the adaptive synthetic inertia and stabilizes the DC bus voltage in case of disturbances;
The proposed approach follows an optimal control design procedure that makes it scalable with guaranteed steady-state and dynamic performances; and

The proposed method eliminates the needs of external PWM modules and reduces the parameter tuning efforts.

The remainder of this paper is organized as follows: Section II presents the designing flow of the proposed FCS-MPC-based control strategy for building-scale DC microgrids, followed by the virtual capacitance control implementation in Section III; Section IV analyzes the stability of the proposed method and presents the simulation case studies; Section V demonstrates the experimental verification; and Section VI concludes the paper.

II. THE PROPOSED CONTROL STRATEGY FOR ISLANDED DC BUILDING MICROGRID

A. PROBLEM FORMULATION

The proposed strategy features optimal and predictive control actions in a fast and reliable manner. By taking certain measurements, the FCS-MPC determines future control inputs by solving an optimization problem over a finite horizon. It integrates the references and predictions of future states of a DC microgrid using the past and present measurements. Firstly, a DC microgrid is formulated into a discrete-time state-space model:

\[ x(k + 1) = Ax(k) + Bu(k) \]  
\[ y(k) = Cx(k) + Du(k) \]

where \( x \) is the \( n \)-dimensional system states \( (x \in \mathbb{X} \in \mathbb{R}^n) \) and \( u \) is the \( m \)-dimensional control inputs \( (u \in \mathbb{U} \in \mathbb{R}^m) \), by which future state variables \( x(k + 1) \) of the microgrid can be predicted. The control process can be further formulated into an optimization problem with the finite input set \( U_M(x) = \{u(0), u(1), \ldots, u(M - 1)\} \) and terminal region \( \mathbb{X}_f: \)

\[ \begin{align*}
\min_u & \{J(x, u)|u \in U_M(x)\} \\
\text{s.t.} & \ \ x(k) \in \mathbb{X}, \ \ \forall k \in \{0, \ldots, M\} \\
& \ u(k) \in U, \ \ \forall k \in \{0, \ldots, M - 1\} \\
& \ x(k + M) \in \mathbb{X}_f
\end{align*} \]  

with a quadratic cost function:

\[ J(x, u) = \sum_{k=0}^{M-1} \xi(x(k), u(k)) + J_f(x(k + M)) \]  

where \( J_f(x(k + M)) \) is the terminal cost that imposes the states’ convergence to references and \( \xi(x(k), u(k)) \) is stage cost that represents the predicted evolution of the states and control inputs [21]. For the sake of a simpler presentation, the proposed method will follow a one-horizon-step prediction. Therefore, when the microgrid reaches an equilibrium point, it satisfies \( x(k + 1) = x(k) = x^* \). In practice, there are finite elements in the control input set \( U_M(x) \), which are essentially the power electronics control signals. Therefore, Eq. (4) will be evaluated by each element of \( U_M(x) \) until the optimal input sequence is obtained. The model moves step by step in the sample horizon as measurements, predictions, the value of cost function update iteratively. Detailed designing procedures are elaborated below.

B. ISLANDED DC MICROGRID MATHEMATICAL MODELING WITH POWER SHARING MECHANISM

Fig. 2 illustrates the schematic of the studied DC building microgrid with \( N \) parallel DERs, which can be renewable generators such as PV array, diesel generator, ESS, etc. It is noteworthy that although the following design process is model-based, it is a unified approach that is adaptable to other types of DC microgrids with proper modifications. To enable the plug and play of loads or DERs in islanded mode, the bus voltage should always be stabilized. Mathematically, the prediction for output current of DER, can be formulated as:

\[ i_{L_i}(k + 1) = [S_iV_i^{in}(k) - V_{bus}(k)][T_s/L_i] + i_{L_i}(k) \]  

where \( S_i \in \{0, 1\} \) is the converter switch status and \( T_s \) denotes the sampling period of the discrete system. Therefore, the DC bus voltage can be formulated by:

\[ V_{bus}(k + 1) = Z_{DC} \sum_{i=1}^{N} i_{L_i}(k + 1) \]

\[ = Z_{DC}T_s\left[\frac{1}{L_1} \ldots \frac{1}{L_N}\right][S_1V_1^{in}(k) - V_{bus}(k) \ldots S_NV_N^{in}(k) - V_{bus}(k)]^T + V_{bus}(k) \]

given the fact that:

\[ Z_{DC} \sum_{i=1}^{N} i_{L_i}(k) = V_{bus}(k) \]
where the instant DC load impedance, $Z_{DC}$, can be determined by real-time measurement of the bus current. By Eq. (6) and (7), the future value of the DC bus voltage can be predicted in a one-step horizon, which will be utilized for bus voltage regulation later.

A current sharing mechanism is implemented in the FCS-MPC model by inserting a current sharing vector

$$ \Gamma = \{ \gamma_1, \gamma_2, \ldots, \gamma_{N-1} \} $$

(8)

which imposes the load sharing among DERs in a predictive fashion. Then, the following term is defined:

$$ i_{\gamma_i}(k+1) = i_{L_i}(k+1) - \gamma_i i_{L_{i+1}}(k+1) $$

(9)

This controls the output current ratio between the $i^{th}$ and $(i+1)^{th}$ DERs. Therefore, from Eq.‘s (5)-(9), the state-space model of the entire $N$-DER DC microgrid can be mathematically represented as:

$$ x_{mg}(k+1) = Ax_{mg}(k) + Bu_{mg}(k) $$

(10)

where the semi-positive definite matrices $Q$ and $R$ and positive definite matrix $P$ are the weighting factors of the predicted behavior and controlled states, respectively. In the practical case presented in this paper, the reference can be set as $x_{mg}^* = [V_{bus}^*, 0 0 \ldots 0]^T$ to impose the output current ratios during the control actions, which will yield $i_{L_1}/i_{L_2} = \gamma_1, i_{L_2}/i_{L_3} = \gamma_2, \ldots, i_{L_{N-1}}/i_{L_N} = \gamma_{N-1}$. For each sampling instant, cost function (11) will be evaluated once and predictive optimal control inputs for the DC microgrid, $u_{mg}^{opt}$, can be obtained, i.e., $u_{mg}^{opt} = \arg\min_{u_{mg}} \{ J(x_{mg}, u_{mg}) \}$.

The results of $J(x_{mg}, u_{mg})$ and $u_{mg}^{opt}$ are updated interactively as the time-series sampling instant moves forward. By properly designing the $P$, $Q$, and $R$, the MPC control loop will exhibit guaranteed and provable performance. It has been proven in [22] that, finding proper weighting matrices for the MPC formulation to guarantee its initial-state practical control Lyapunov stability is equivalent to solving the following Algebraic Riccati equation:

$$ A^T_K P A_K - P + Q + K^T R K = 0 $$

(12)

where

$$ K = -(B^T P B + R)^{-1} B^T P A $$

(13)

$$ A_K = A + BK $$

(14)

By defining the maximum input $u_{max}$, the nominal control set can be defined as:

$$ U \triangleq \{ u \in \mathbb{R}^n : |u| \leq |u_{max}| \} $$

(15)

Then, the maximum quantization error of the control input $\Delta_q$ can be obtained. The terminal region $X_f$ can be characterized by:

$$ X_f \triangleq \{ x_{mg} \in \mathbb{R}^n : |x_{mg} - x_{mg}^*| \leq \frac{|u_{max}|}{|R|} \} $$

(16)

The cost function Eq. (11) can be considered as a practical control Lyapunov function, satisfying that, for all states $x \in X_M$, where $X_M$ is the domain of the cost function, the following conditions hold:

$$ J(x) > \alpha_1 |x|^2, \forall x \in X_M $$

(17)

$$ J(x) < \alpha_2 |x|^2, \forall x \in X_f $$

(18)

$$ \Delta J(x) < -\alpha_3 |x|^2 + |B^T P B + R| \Delta_q $$

(19)

where $\alpha_1 = \lambda_{min}(P)$, $\alpha_2 = \lambda_{max}(P)$, $\alpha_3 = \lambda_{min}(Q)$. The decaying factor, denoted by $\rho$, can be calculated by:

$$ \rho = 1 - \alpha_3/\alpha_2 $$

(20)

and the ultimately bounded set $X_b$ for the state variables can be found by:

$$ X_b \triangleq \{ x_{mg} \in \mathbb{R}^n : |x_{mg} - x_{mg}^*| \leq \frac{|B^T P B + R|}{\alpha_1 (1-\rho)} \Delta_q \} $$

(21)

The region of attraction $X_{MPC}$, which is defined as:

$$ X_{MPC} \triangleq X_f \cup \{ x_{mg} \in \mathbb{R}^n : \Delta J(x_{mg}) < 0 \} $$

(22)

can be obtained by enlarging $X_f$ while guaranteeing that $\Delta J(x_{mg}) \leq 0$. 

C. CONTROLLER DESIGN WITH PERFORMANCE GUARANTEES

Future state values of the DC microgrid, $x_{mg}(k+1)$, can therefore be predicted by Eq. (10) using present state measurements $x_{mg}(k)$ and tentative control actions $u_{mg}(k)$. Consequently, DC bus voltage regulation and power sharing mechanism can be achieved simultaneously by controlling $x_{mg}(k+1)$ to track the designated reference $x_{mg}^*$ via minimization of the following cost function at each sampling step:

$$ J(x_{mg}, u_{mg}) = |x_{mg}(k+1) - x_{mg}^*|^T P |x_{mg}(k+1) - x_{mg}^*| $$

$$ + |(x_{mg}(k) - x_{mg}^*)|^T Q |x_{mg}(k) - x_{mg}^*| $$

$$ + |u_{mg}(k) - u_{mg}^*|^T R |u_{mg}(k) - u_{mg}^*| $$

(11)
III. VIRTUAL CAPACITANCE CONTROL

In addition to guarantee the initial-state practical stability, virtual capacitance control is proposed for stabilizing the DC bus voltage in transient scenarios. The proposed method is elaborated in this section. Due to high penetration of power-electronics DERs, DC microgrid systems in general exhibit extremely low inertia phenomenon and the bus voltage is vulnerable to abrupt load or renewable changes. There are researches seeking for methods to provide virtual inertia support for AC microgrids to maintain a stable frequency, for instance, by active power injection or VSG. Nevertheless, in DC microgrids, instead of frequency stability, the lack of system inertia can lead to bus voltage instability. To the best of authors’ knowledge, DC microgrid virtual capacitance control with FCS-MPC has not been considered in the literature. To address this issue, MPC-based virtual capacitance control is proposed to enhance the synthetic inertia of DC microgrids. This is achieved by synthesizing the effect of a virtual capacitor across the DC bus to suppress voltage fluctuations. Instead of directly using the static voltage reference \( V_{bus}^* \) from upper-level EMS, the controller hypothesizes that there is an ultra-capacitor \( C_V \) across the DC bus that is absorbing a virtual current as:

\[
    i_{VC} = C_V \frac{dV_{bus}(t)}{dt} = C_V \frac{V_{bus}(k) - V_{bus}(k-1)}{T_s} \tag{23}
\]

As is illustrated in Fig. 2, the virtual capacitor yields the current fed to the DC loads to be:

\[
    i_{iLoad} = \sum_{i=1}^{N} i_{Load} - i_{VC} \tag{24}
\]

Therefore, from (23) and (24) and by taking into account the dynamic effect of virtual capacitor, the synthetic bus voltage reference is given by:

\[
    V_{VC}^{*} = Z_{DC} \cdot \frac{i_{VC}}{i_{Load}} = V_{bus}^{*} - \frac{V_{bus}(k)}{i_{Load}(k)} \cdot C_V \cdot \frac{V_{bus}(k) - V_{bus}(k-1)}{T_s} \tag{25}
\]

which is adaptive to the dynamics of bus voltage \( V_{bus} \). Fig. 3 illustrates an outline of the proposed strategy. When there are load perturbations, e.g., due to load or power fluctuations, the virtual capacitor will buffer the extra amount of current and dynamically adjust the value of \( V_{bus}^{*} \) to smooth the DC bus voltage. For instance, when DC load suddenly increases, \( V_{bus} \) will decrease, which yields two successive voltage measurements \( V_{bus}(k) < V_{bus}(k-1) \) and results in \( V_{VC}^{*} \) to increase dynamically as per Eq. (25). The controller notices the change of reference and will force \( V_{bus}^{*} \) to track the new \( V_{VC}^{*} \) with the best effort, which reduces the dip of bus voltage \( V_{bus} \). As \( V_{bus} \) goes back on the track, \( V_{VC}^{*} \) converges to \( V_{bus}^{*} \) when \( V_{bus}(k) = V_{bus}(k-1) \). Consequently, the bus voltage will be stabilized to EMS reference \( V_{bus}^{*} \) in steady state. This process is shown in Fig. 4, where the blue and red curves are \( V_{VC}^{*} \) and \( V_{bus}^{*} \), respectively.

IV. SIMULATION CASE STUDIES

A. SYSTEM SETUP

To validate the proposed control scheme, a building-scale DC microgrid is modeled in the PSCAD/EMTDC software package. The microgrid under investigation shares the same configuration as Fig. 2 with three parallel DC DERs on the DC bus. The microgrid operates in islanded mode without utility power exchange. Detailed parameters of the tested system are presented in Table 1. For a clearer comparison, per-unit values are used, with voltage base 1 kV, current

![Figure 3: The proposed control strategy for DC Microgrid.](image)

![Figure 4: Dynamic response of the virtual-capacitance-enabled DC bus voltage \( V_{bus} \) and reference \( V_{VC}^{*} \) during an abrupt load change.](image)

**TABLE 1. Parameters of the Case Study System**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage of DER₁</td>
<td>( V_{1in} )</td>
<td>2 pu</td>
</tr>
<tr>
<td>Voltage of DER₂</td>
<td>( V_{2in} )</td>
<td>2 pu</td>
</tr>
<tr>
<td>Voltage of DER₃</td>
<td>( V_{3in} )</td>
<td>2 pu</td>
</tr>
<tr>
<td>Output Filter Inductor of DER₁</td>
<td>( L_1 )</td>
<td>1 mH</td>
</tr>
<tr>
<td>Output Filter Inductor of DER₂</td>
<td>( L_1 )</td>
<td>1 mH</td>
</tr>
<tr>
<td>Output Filter Inductor of DER₃</td>
<td>( L_1 )</td>
<td>1 mH</td>
</tr>
<tr>
<td>Current Sharing Ratio 1</td>
<td>( \gamma_1 )</td>
<td>1.2</td>
</tr>
<tr>
<td>Current Sharing Ratio 2</td>
<td>( \gamma_2 )</td>
<td>1.2</td>
</tr>
<tr>
<td>Desired DC Bus Voltage</td>
<td>( V_{bus} )</td>
<td>1 pu</td>
</tr>
<tr>
<td>Sampling Period</td>
<td>( T_s )</td>
<td>80 us</td>
</tr>
<tr>
<td>Initial DC Load Impedance</td>
<td>( Z_{DC} )</td>
<td>1 pu</td>
</tr>
</tbody>
</table>
base 1 kA, impedance base 1 Ω, and power base 1 MW. It is noteworthy that the proposed method follows a unified designing approach which is scalable and can be applied to other DC microgrids in different configurations via proper modifications. Multiple case studies are performed and the results are analyzed below.

Following the proposed procedures in Section II, state-space model of the DC microgrid with three DERs can be derived using Eq. (10) with the parameters given in Table 1. For the sake of simplicity, weighting matrices in the cost function (11) are selected as $Q = I_{3\times3}$ and $R = 1e^{-3}I_{3\times3}$. By solving the Algebraic Riccati equation (12), matrix $P$ can be calculated as:

$$
P = \begin{bmatrix}
1.0296 & 0.0074 & 0.0074 \\
0.0074 & 1.0783 & 0.0371 \\
0.0074 & 0.0371 & 1.0783
\end{bmatrix} \tag{26}
$$

while matrix $K$ can be obtained as:

$$
K = \begin{bmatrix}
-3.6851 & -6.8520 & -3.5539 \\
-2.9711 & 3.9622 & -3.2936 \\
-2.3761 & 2.9698 & 6.9275
\end{bmatrix} \tag{27}
$$

Iterations of cost function $\mathcal{J}$ are depicted in Fig. 5, which shows the minimization process of Eq. (11). When the system starts, $\mathcal{J}$ begins to decay from a relatively large number. Within 2 ms, the cost of controller converges to a value near zero, which indicates that the state variables are tracking their references and the microgrid is stable.

**B. CASE STUDY 1 - VIRTUAL CAPACITANCE CONTROL**

The first case aims at verifying the performance of the proposed FCS-MPC virtual capacitance control. Fig. 6 presents the transient behaviors of the DC bus voltages in response to sudden load changes with a desired voltage of 1 pu. Comparisons are carried out with the virtual capacitance varying from 0 to 150 mF with an increment of 50 mF. At $t = 2$ s, the load impedance at DC bus decreases from 1 pu to 0.8 pu, which results in a nuisance voltage disturbance. Without $C_V$, the voltage decreases by more than 3% (black curve). However, with the synthetic capacitance, the dip of voltage resulted from load change can be mitigated to a value within the ±1% zone in 200 ms (zoomed area in Fig. 6). As the value of $C_V$ increases, the synthetic capacitor current ($i_{VC}$ in Eq. (23)) increases and so does the inertia control effect to the DC bus. At $t = 3$ s, the load impedance increases from 0.8 pu to 1 pu and the response of bus voltage verifies how the perturbation is reduced by the virtual capacitance again. This is advantageous in practice. When there is a severe disturbance that leads to voltage violations of the DC bus, the proposed virtual capacitance control technique can increase the system inertia and allow more time for the EMS to restore the bus voltage and avoid interruptions to critical loads.

**C. CASE STUDY 2 - LOAD FLUCTUATION RESPONSE**

Since loads in a DC building can vary arbitrarily during daily operations, the DC bus voltage may suffer from frequent fluctuations. To further highlight the promising performance of the proposed scheme, a benchmarking study is carried out by result comparison between the proposed method and the one introduced in a recent publication, Ref. [20], in which a virtual inertia control strategy for DC microgrid is proposed to restrain the DC bus voltage fluctuation. The compared method utilizes a AC/DC interlinking converter (IC) and devises a control strategy to analogize the behavior of a VSG in AC systems. Thereby, the DC bus voltage can be maintained by the IC. A current feed-forward control loop is employed to enhance its dynamic performance. Although this method restrains the voltage fluctuations, steady-state control error cannot be eliminated due to its mechanism. This can be seen in both the reference and the following case study. For comparative studies, the control method in [20] is implemented in a DC microgrid with the same configuration.
and DER parameters. Fig. 7 shows the bus voltage behavior when load fluctuates intensively during a short period. As is depicted, when loads fluctuate irregularly (black curve), the bus voltage controlled by [20] can be smoothed to a certain extent. However, it unavoidably suffers from continuous voltage deviation (red dotted curve). Moreover, the situation worsens when the change of load becomes more significant. This degrades the power quality and may cause sensitive loads to trip when the bus voltage exceed the safety margin. On the contrary, the DC microgrid regulated by the proposed method exhibits satisfactory bus transient dynamics, which eliminates the voltage deviations while alleviating the fluctuations (red solid curve).

**D. CASE STUDY 3 - POWER SHARING CONTROL AND BUS VOLTAGE STABILIZATION**

This case focuses on validating the proposed method for its accurate and flexible power sharing control mechanism among the DERs as well as the voltage dynamics during these transitions. Initially, the current sharing vector $\Gamma = \{\gamma_1, \gamma_2\}$ is set as $\gamma_1 = \gamma_2 = 1.2$, which yields the power provided by the DERs to be $P_{\text{DER1}} = 1.2 \times P_{\text{DER2}} = 1.44 \times P_{\text{DER3}}$. As Fig. 8 shows, before $t = 3$ s, the entire DC load power is 1.1 pu, which is accurately shared by three DERs with 0.436 pu, 0.362 pu, and 0.302 pu, respectively. At $t = 3$ s, the DC load increases to 2.1 pu by a step change of 1 pu. The contribution of each DER increases accordingly by 0.395 pu, 0.330 pu, and 0.275 pu, respectively, resulting in a steady-state power sharing of 0.831 pu, 0.692 pu, and 0.577 pu. Finally, the load declines to 1.35 pu at $t = 5$ s, resulting in a power sharing of 0.534 pu, 0.445 pu, and 0.371 pu. From the testing results showed in Fig. 8, the proposed method is capable of controlling the power sharing ratio of the DERs accurately. The second plot of Fig. 8 illustrates the comparison of bus voltage dynamic responses controlled by the proposed method with virtual capacitance, controller proposed in [20], and conventional PI-based droop control. During these transitions, the proposed method imposes the bus voltage to track the reference smoothly with acceptable disturbances (red solid curve in the voltage plot), while the other two methods suffer from bus voltage deviations (blue and green solid curves).

Moreover, the power sharing ratio vector $\Gamma = \{\gamma_1, \gamma_2\}$ can be adapted flexibly during real-time operation according to the commands from EMS. This is an important feature for islanded building-scale microgrids as the EMS should be capable of managing the outputs of each DER as per their real-time capacity and availability. In practice, the building EMS can utilize the proposed method to adjust the output powers of multiple energy storages according to their real-time SOCs during operation. To validate this, additional case studies are performed and the results are depicted in Fig. 9. Initially, the power sharing ratio is set as $\gamma_1 = \gamma_2 = 2$. Power outputs of the three DERs follow the ratio accurately as can be observed from the plot. At $t = 2$ s, the ratio is suddenly changed to $\gamma_1 = \gamma_2 = 1$, which yields an identical output power from each DER. Afterwards, $\Gamma$ is set as $\gamma_1 = \gamma_2 = 2$ at $t = 4$ s and $t = 6$ s, respectively. As is shown in Fig. 9, the outputs of DERs are able to follow the command in an accurate and timely manner. During these transitions, the DC bus is maintained at a stable voltage (black solid curve).

**E. CASE STUDY 4 - DER PLUG AND PLAY**

Plug-and-play capability of an islanded DC microgrid is essentially important due to the absence of utility grid. This case aims at verifying the performance of proposed control strategy in terms of DER plug and play. During operation, a microgrid may lose one or more of its DERs due to converter or communication failures. After cleaning the faults, the lost DER will be reconnected to the system. The controller must be able to handle this contingency by maintaining the bus voltage and redistribute the power among healthy DERs. In this case study, DER$_3$ was disconnected by a breaker at $t = 5$ s. As Fig. 10 depicts, the outputs of DER$_1$ and DER$_2$ increase simultaneously following the sharing ratio $\gamma_1 = 4$. At $t = 9$ s, the fault is cleared and DER$_3$ is reconnected to the bus. The controller successfully redistributed the power sharing...
Therefore, in practice, the proposed method will allow the DC bus, resulting in a slower response to the step change. The value of virtual capacitance will increase the inertia of the system. As shown in Fig. 11, increasing the virtual capacitance, the bus voltage follows the reference within a superior steady-state accuracy. With a proper value of virtual capacitance, the convergence of dynamic tracking error and provides a superior steady-state accuracy. With a proper value of virtual capacitance, the bus voltage follows the reference within a satisfactory dynamic response. Note that PI-based controllers generally require extensive parameter tuning, while the proposed method significantly reduces this effort thanks to its mechanism. The case study also demonstrates the influence of a large virtual capacitor. As shown in Fig. 11, increasing the value of virtual capacitance will increase the inertia of DC bus, resulting in a slower response to the step change. Therefore, in practice, the proposed method will allow the change of $C_V$ value during operation. For example, further algorithms can be developed on top of the proposed mechanism to make $C_V$ adaptive, so the desired dynamics of bus voltage will become flexible.

F. CASE STUDY 5 - BUS VOLTAGE CHANGE

This case provides verification of the proposed FCS-MPC approach in controlling the DC bus voltage in response to step changes of reference, compared with conventional PI-based controllers. The bus voltage reference changes from 1 pu to 0.95 pu at $t = 2$ s and back to 1 pu at $t = 3$ s. Fig. 11 shows the dynamic response of the measured bus voltage controlled by the proposed method with small (20 mF) and large (150 mF) virtual capacitance (red and orange curve, respectively) and conventional PI controller (blue curve). The proposed optimal-control-based approach guarantees the convergence of dynamic tracking error and provides a superior steady-state accuracy. With a proper value of virtual capacitance, the bus voltage follows the reference within a satisfactory dynamic response. Note that PI-based controllers generally require extensive parameter tuning, while the proposed method significantly reduces this effort thanks to its mechanism. The case study also demonstrates the influence of a large virtual capacitor. As shown in Fig. 11, increasing the value of virtual capacitance will increase the inertia of DC bus, resulting in a slower response to the step change. Therefore, in practice, the proposed method will allow the change of $C_V$ value during operation. For example, further algorithms can be developed on top of the proposed mechanism to make $C_V$ adaptive, so the desired dynamics of bus voltage will become flexible.

V. EXPERIMENTAL CASE STUDIES

To further validate the proposed method, a laboratory DC microgrid prototype is set up and multiple real-time experimental tests are performed. Fig. 12 and 13 present the system configuration and the prototype test-bench, respectively. The DC microgrid consists of two DERs and multiple DC loads on a common bus. Two DC/DC buck converters are used to interface the DERs. dSPACE MicroLabBox (DS1202) hardware-in-the-loop (HIL) platform is employed to integrate the test-bench and the proposed controller that is implemented in the host server using Matlab/Simulink and dSPACE ControlDesk software packages. Both DER voltages ($V_{1in}$ and $V_{2in}$) are set as 24 V and the DC bus voltage ($V_{bus}$) is regulated at 10 V.
The current sharing ratio is set as $\gamma_1 = \frac{1}{2}$, and dSPACE sampling period is $T_s = 100$ us. The weighting matrices are chosen as $Q = I_{2 \times 2}$ and $R = 0_{2 \times 2}$. By Eq.'s (14) and (15), one can calculate $P = I_{2 \times 2}$ and

$$K = \begin{bmatrix} -0.6 & 20 \\ 1.8 & -20 \end{bmatrix}$$

(28)

A. CASE STUDY 1 - POWER SHARING MECHANISM

Firstly, performance of the power sharing mechanism and bus voltage regulation is verified. As Fig. 14 depicts, loads on the DC bus draw a total current ($i_{\text{Loads}}$) of 0.8 A at the beginning of testing, which is distributed by the two DERs with approximately $i_{L1} = 0.25$ A and $i_{L2} = 0.55$ A, which complies with $\gamma_1 = \frac{1}{2}$. At around 9.6 s, a sudden load change increases $i_{\text{Loads}}$ to 0.84 A. As a result, the contributions of DERs, $i_{L1}$ and $i_{L2}$, increase to 0.58 A and 0.26 A at a ratio of around 2.2. At 15.4 s, loads decrease and the contributing currents of both DERs decline accordingly. Fig. 14(a) illustrates the real-time current measurements captured by the dSPACE ControlDesk. During these processes, the DC bus voltage ($V_{\text{bus}}$) is regulated at 10 V stably, with slight disturbances during the transitions (oscilloscope screenshot in Fig. 14(b)).

B. CASE STUDY 2 - VIRTUAL CAPACITANCE CONTROL

The second case evaluates the virtual capacitance control of the DC bus. Comparisons are carried out by measuring the dynamics of $V_{\text{bus}}$ during the transition of load step change from 10 to 8.5 Ω. Fig. 15 shows the oscilloscope screenshots, where (a) is the voltage disturbance without virtual capacitance control and (b) is with virtual capacitance $C_V = 10$ mF. As is measured by the cursors, the undershoot of $V_{\text{bus}}$ reaches 9.225 V without virtual capacitance control. However, with $C_V = 10$ mF, the undershoot of $V_{\text{bus}}$ is improved to 9.635 V, which further validates the effectiveness of the proposed method.

VI. CONCLUSION

This paper presents a novel control strategy for islanded building-scale DC microgrids, which eliminates the DC bus voltage deviations under system disturbances. A new DER power sharing mechanism is proposed in a predictive fashion by introducing a current sharing vector in the MPC model of DC microgrid. Moreover, virtual capacitance control is implemented in the MPC to provide adaptive synthetic inertia support. The state-space modeling procedures of DC microgrids are presented, based on which the control scheme is devised in an optimal control basis with Lyapunov stability performance guarantees. The controller predicts future behaviors of state valuables, and, by minimizing a cost function with certain practical constraints, offers optimal actions for the DERs to track EMS references. Both simulation and experimental case studies with benchmarking comparisons and in-depth analyses are carried out. Results verify the
promising performance of the proposed strategy.

REFERENCES


