A cooperative game approach for coordinating multi-microgrid operation within distribution systems

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HIGHLIGHTS

- A coalitional operation model for multiple microgrids to achieve global optimum.
- A cost allocation method from cooperative game theory to achieve local optimum.
- A linearized optimal power flow with voltage constraints to realize cooperation.
- The economy benefits of multi-microgrid cooperation are simulated and analyzed.

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ABSTRACT

This paper focuses on simulating the potential cooperative behaviors of multiple grid-connected microgrids to achieve higher energy efficiency and operation economy. Motivated by the cooperative game theory, a group of individual microgrids is treated as one grand coalition with the aim of minimizing the total operation cost. Next, given that each microgrid operator is an independent and autonomous entity with the aim of maximum self-interest, a cost allocation method based on the concept of core in the cooperative game is implemented to ensure a fair cost share among microgrid coalition members, which guarantees the economic stability of the coalition. Considering the combinatorial explosive characteristic of the cost allocation problem, Benders Decomposition (BD) algorithm is applied to locate the core solution with computational efficiency. In addition, since microgrid coalition is formed at the distribution system level, network losses is not negligible. After considering network losses, the coalition operation model of multi-microgrid becomes an optimal power flow problem. A linearized optimal power flow for distribution (LOPF-D) model is applied instead of the conventional ACOPF model to reduce computation burden, meanwhile maintaining adequate accuracy. Case studies on standard IEEE systems demonstrate the advantages of multi-microgrid cooperation and the robustness of the formulated grand coalition. In addition, comparisons with the conventional ACOPF model verifies the high performance of the proposed LOPF-D model.

1. Introduction

The worldwide energy and environmental crisis has led to the large-scale development of renewable energy sources (RES) and distributed energy resources (DER), which in return has brought microgrid technology under spotlight in the power industry. A microgrid is a small-scale electric power system which contains distributed resources and load, and can operate in either grid-connected mode or islanded mode [1]. Currently, emerging new types of demand-side resources have been spotted in microgrids, including electrical vehicles, air conditioning loads, and refrigerators, which add considerable flexibility to microgrid operation. The advantages of integrating microgrids into distribution systems are multifold: first, the DER units that reside in a microgrid can support the local energy demand, hence reduces its reliance on the upper-level utility grid and enhances the reliability of power supply; second, it follows that microgrid facilitates environmentally-friendly energy consumption by utilizing renewable energy-fueled generators, i.e., wind turbines, photovoltaic panels, and fuel cells; last but not least, by supplying the energy demand via local distributed generators (DGs), microgrid can reduce long-distance transmission loss, as well as the
investment on large-scale transformers and transmission lines.

Most recently, with the increasing penetration of renewable energy into power systems, the concept of multi-microgrid (MMG) comes up on the stage, which refers to a cluster of microgrids connected with each other in close electrical or spatial distance [2,3]. The aim of MMG is to achieve resilience and stability via fast power exchange and to further obtain a high and smooth penetration of DERs into the bulk system. Possible architectures for multi-microgrid regarding layout and inter-faces accompanied by cost and reliability analysis are discussed in [4]. To achieve a coordinated penetration of multi-microgrid into the bulk power system, a hierarchical control strategy is proposed in [5,6], which includes the primary droop-control of power electronic devices, the secondary control for voltage/frequency restoration and synchronization, and the tertiary control of real and reactive power. The last one is in association with microgrid energy management system, and can be formulated as an economic dispatch problem with the aim of maximizing economic profit.

The focus of this paper lies in the tertiary control level of a multi-microgrid system. In retrospect, existing works mainly cover two topics related to this field: planning and operation. In terms of the former, Ref. [7] applies the Decision-Tree (DT) method to plan the capacities of energy storage devices within microgrids to realize local power balance; Ref. [8] includes the coupling physical and operational constraints of electrical and heating/cooling networks for multi-energy microgrids in the design of the capacities of DER units; Ref. [9] combines both DER sizing and placement problems into one mixed integer linear programming, where microgrid is modeled as a multi-node system instead of an aggregated single-node model to better consider power flow and heat flow balances.

With regard to operation, existing research works mainly adopt two approaches to coordinate MMG economic dispatch: the centralized approach and the decentralized approach. The main idea behind the centralized optimization method is to aggregate all the entities into the system as one unity with a collective objective. In the case of multi-microgrid coordination, a central controller is selected (i.e. distribution system operator, DSO) to organize the operation of all the DGs and loads regardless of their individual interests. In this aspect, Ref. [10] establishes a centralized control model of a group of microgrids that can exchange power with their neighbors, where the objective is to maximize the total profit of all microgrid operators. Simulation results indicate that local energy exchange improves individual operation economy by making full use of the zero-cost renewable energy. In [11], the interactions between the upper-level distribution system and the multi-microgrid system are further considered, and the DSO is included as an additional independent entity in MMG coordination. To decrease model complexity and improve computational efficiency, decentralized dispatch methods have been applied in [12,13], where the global optimization model is decomposed into several independent sub-problems using Lagrange relaxation method and solved by local entities. Model predictive control (MPC) scheme is implemented in [14,15] in a distributed manner to address the uncertainties of load and renewable energy within the microgrids and to maintain a steady power exchange with the rest of the distribution system. The authors in [16,17] explore the optimal risk-constrained bidding strategies of microgrids for providing ancillary service to the utility grids using decentralized and centralized approaches, respectively.

There exist some challenges with the above conventional models [18]: in the centralized method, since it requires full communication among all entities within the entire network, it is not scalable, especially not suitable for plug-and-play DERs like electrical vehicles; in the decentralized method, since local entities independently work on their own optimal dispatch schedule without the information from other entities, this complete isolation from the rest of the system usually cannot reach global optimum. In summary, the centralized method has a simple implementation to realize global optimum, while the decentralized method focuses on local optimum. Nevertheless, there remains some gap between the two goals, which may sabotage the coordinated operation of the multi-microgrid. The reason is that each microgrid is a highly independent and profit-driven entity with the goal of maximizing its self-interest. Thus conflicts of interests between the local microgrid (local optimum) and the system operator (global optimum) may drive microgrid away from coordination.

The motivation of this work is to address the above mentioned concerns between global optimum and local optimum. In this paper, we
proposes a cooperative game approach to implement multi-microgrid coordinated operation. Non-cooperative game and cooperative game are the two fundamental pillars of game theory. Both games intend to reach a globally balanced status where no player can get any further improvement of their interests, which is referred to as a Nash Equilibrium (NE) in the former, and a core status in the latter. However, the difference between the two lies in that the non-cooperative game focuses on obtaining the maximum individual payoff for each single player without evaluating the global welfare. While in the cooperative game, a coalitional optimization model is first developed to reach the global optimum, then a cost or profit allocation model is established to fairly distribute the collective benefits among all the players to guarantee local optimum. Hence, it can be safely concluded that the cooperative game approach is a natural fit for multi-microgrid cooperation problem to mitigate the potential interest collisions between global and local stakeholders.

In retrospect, considerable efforts have been made in implementing cooperative game into the field of power system, from transmission cost allocation [19–21] to revenue distribution among the portfolio of power generators [22,23]. The Shapley value from cooperative game theory is applied in [24] to measure the flexibility of the fast-ramping DERs. However, application of cooperative game in multi-microgrid coordination is still at its initial stage of research, which leaves great potentials for further explorations. In this regard, the authors in [25,26] treat each microgrid as an active cooperative player seeking for potential coalitions with their neighbors to share power and save transmission cost. A merge-and-split algorithm is developed to guide the formation of different coalitions under environmental changes. The advantages of direct power exchange among local DERs and consumers are discussed in [27], and the Shapley value is proposed as the optimal cost saving division among the players, which belongs to the set of core solution. A cooperative generation planning model for interconnected microgrids is proposed in [28,29], in which both the long-term investment cost and short-term operational cost are included in the fair cost distribution model. A Nash bargaining solution is implemented as extra earnings or total cost among the coalition members[32]. A cooperative game, or coalitional game, is the study concerned with the group of rational players who coordinate their actions and pool their winning, which consequently leads to the problem of how to divide the extra earnings or total cost among the coalition members [32]. A cooperative game consists of two essential elements: (1) a set of players \( N = \{1,2,\ldots,i,\ldots,n\} \) and (2) a characteristic function \( \nu \) that specifies the value created by different subsets of the players. A coalition \( c \) refers to a subset of the players. The grand coalition includes all the players. An allocation \( x \) is a way to distribute the value created by grand coalition, marked as \( \nu(1) \), among all the players. Several other related definitions are listed as follows:

2. Coalitional operation model of multi-microgrid system

2.1. A brief introduction of cooperative game theory

Cooperative game, or coalitional game, is the study concerned with a group of rational players who coordinate their actions and pool their winning, which consequently leads to the problem of how to divide the extra earnings or total cost among the coalition members [32]. A cooperative game consists of two essential elements: (1) a set of players \( N = \{1,2,\ldots,i,\ldots,n\} \) and (2) a characteristic function \( \nu \) that specifies the value created by different subsets of the players. A coalition \( c \) refers to a subset of the players. The grand coalition includes all the players. An allocation \( x \) is a way to distribute the value created by grand coalition, marked as \( \nu(1) \), among all the players. Several other related definitions are listed as follows:

![Diagram](image-url)  
*Fig. 1. IEEE 33-bus distribution system with 10 microgrids.*
(1) an allocation \((x_1, x_2, \ldots, x_n)\) is individually rational if \(x_i \geq v(i)\) for all \(i\); (2) an allocation \((x_1, x_2, \ldots, x_n)\) is efficient if \(\sum x_i = v(1)\); (3) an allocation \((x_1, x_2, \ldots, x_n)\) is coalitionally rational if \(\sum x_i \geq v(c)\) for all the subsets \(c\); (4) an allocation \((x_1, x_2, \ldots, x_n)\) is said to lie in the core of the game if it satisfies all three conditions above.

From the above definitions, it can be discovered that a core allocation ensures that every player in the grand coalition benefits more than in the case when they each work alone or form coalitions. Thus, no one would be willing to leave the grand coalition and its stability can be ensured.

In the case of multi-microgrid coordination, the players are individual microgrid operators, and all players are assumed to automatically form a grand coalition with ex-ante binding contracts. The associated characteristic function is the total operation cost. The focus of the problem is to allocate the total line power \(T_{tP}\) and \(T_{tc}\) stand for the total line power of the operation model of multi-microgrid grand coalition, and Section 4 will center on tackling the fair cost allocation problem.

2.2. A coalitional operation model of multi-microgrid system

2.2.1. Objective function

A typical network topology of distribution system with 10 micro-grids integrated is shown in Fig. 1. In this figure, each microgrid contains different types of DGs on the generation side, including micro turbines (MT), boilers, thermal energy storage (TES), wind turbines and photovoltaic solar panels, and electrical load and thermal load on the demand side. All microgrids form one grand coalition based on binding contract, and their objective is to minimize the total operation cost:

\[
\min c = \sum_{i=1}^{N_f} \left( p_i(t)P_{gild}(t) + p_0(t)Q_{qild}(t) + \sum_{m \in \{\sum_{MT=1}^{N_MT} C_{fuel}P_{MT,m}(t) / \eta_{MT} + \sum_{b \in \{\sum_{B=1}^{N_B} C_{fuel}P_{BM,b}(t) / \eta_{B}\}}\} \right) \tag{1}
\]

Eq. (1) calculates the total operation cost of microgrid coalition \(c\), in which the first two terms are the cost for exchange active and reactive power of power exchange with the transmission system. The power exchange is evaluated as the power flow at the point of common coupling (PCC) shown in Fig. 1. \(N_f\) is the length of operation cycle, in this study it is set to 24 h. The following two terms are the generation cost of microgrid m.

2.2.2. Constraints

Operation of microgrid coalition \(c\) should obey the following constraints to ensure system-wide economy and stability:

(1) The scale of microgrid coalition:

\[0 < \text{Fe} < M\] \tag{2}

In Eq. (2), \(M\) is the total number of microgrids, \(c\) is an \(M \times 1\) vector composed of 0–1 binary indices \([c_1, c_2, \ldots, c_m, \ldots, c_M]^T\), where \(c_m\) indicates whether the \(m^{th}\) microgrid belongs to the coalition \(c\) or not. It is obvious that in the grand coalition case, we have \(1^T c = M\).

(2) Capacity of DGs:

\[c_{\text{MT}, \min} \leq P_{MT,m}(t) \leq c_{\text{MT}, \max}\] \tag{3}

\[c_{\text{B}, \min} \leq P_{BM,b}(t) \leq c_{\text{B}, \max}\] \tag{4}

Eqs. (3) and (4) are the capacity constraints of DGs, where \(c_{\text{MT}, \min}\) and \(c_{\text{MT}, \max}\) are the lower bound and upper bound, respectively, of the generator output. If the \(m^{th}\) microgrid and its DGs does not belong to coalition \(c\), \(c_m\) is 0, and \(P_{(MT,B),m}(t)\) equals 0. Otherwise we have \(P_{(MT,B),m}(t) \leq P_{(MT,B),m}^{\text{max}}\).

(3) Charge/discharge constraints of thermal energy storage:

\[S_{\text{in}, m}(t) = S_{\text{in}, m}(t-1) - \Delta P_{\text{in}, m}(t)/\eta_{\text{in}}\] \tag{5}

\[S_{\text{in}, \min} \leq S_{\text{in}, m}(t) \leq S_{\text{in}, \max}\] \tag{6}

\[P_{\text{in}, m}(t) \leq c_mP_{\text{in}, m}^{\text{max}}, P_{\text{in}, m}(t) \geq -c_mP_{\text{in}, m}^{\text{max}}\] \tag{7}

\[S_{\text{in}, m}(N_f) \geq S_{\text{in}, m}(0)\] \tag{8}

Eq. (5) is the inter-temporal constraint of energy level in the thermal storage. For simplicity, we assume that the charging and discharging efficiency of the storage are the same. Eq. (6) implies that the energy of the storage belongs to the microgrid coalition \(c\) or not. Eq. (8) requires that the energy level at the end of the operation cycle should be no lower than its initial value.

(4) Power balance constraint:

\[P_{gild}(t) + \sum_{MT=1}^{N_MT} P_{MT,m}(t) + \sum_{B=1}^{N_B} c_m P_{BM,b}(t) + \sum_{i=1}^{N_i} c_i P_{i}(t) = 0\] \tag{9}

Eqs. (9) and (10) are the active and reactive power balance constraint of the distribution system, where \(P_{\text{load}}(t)\) and \(Q_{\text{load}}(t)\) are the network losses. Since both terms are nonlinear and nonconvex functions of the other control variables, i.e. generator output, which adds great model complexity. We further apply a linearized optimal power flow for distribution (LOPF-D) model to overcome this computation difficulty. More details of the LOPF-D model will be revealed in Section 3.

(5) Bus voltage constraint:

\[V_{ij}(t) = V_{ij}(t-1) - (P_{li}(t) + P_{li}^{\text{max}}(t))R_i + (Q_{li}(t) + Q_{li}^{\text{max}}(t))X_i)/V_{ij}(t)\] \tag{11}

\[V_{ij}(t) = 1.05 \text{ p.u.}\] \tag{12}

\[V_{\text{min}} \leq V(t) \leq V_{\text{max}}\] \tag{13}

Eqs. (11)–(13) is based on the line model illustrated in Fig. 2 [33]. The line that has a tail bus \(k + 1\) is numbered as \(L_k\). For a line that connects a head bus \(i\) to a tail bus \((k + 1)\), the relationship between bus voltage \(V_{ij}(t)\) and \(V_{ij}(t)\) are shown in Eq. (11), where \(P_{li}(t) + P_{li}^{\text{max}}(t)\) and \(Q_{li}(t) + Q_{li}^{\text{max}}(t)\) stand for the total line power flow from bus \(i\) to

\[V(t) + jQ_i(t) = V(t + 1)(t)\]

\[P_{li}(t), Q_{li}(t) = P_{li}(t) + jQ_i(t)\]

\[P_{li}(t) - P_{li}^{\text{max}}(t) + j(Q_{li}^{\text{max}}(t) - Q_{li}^{\text{max}}(t))\]

Fig. 2. Line model in distribution system.
bus $j$. In addition, the vertical voltage drop is neglected, and the head bus voltage is assumed to be close to the rated value.

$$\sum_{m \in \mathcal{E}} (v_h P_{k,s,m}(t) + P_{k,m}(t) + P_{h,m}(t) + c_m P_{solar,m}(t)) - \sum_{m \in \mathcal{E}} c_m P_{load,m}(t) = 0$$

In Eq. (14), $\psi_h$ is the heat to electricity ratio of micro turbines, $P_{solar,m}(t)$ is the heat generated by solar energy, and $P_{load,m}(t)$ is the thermal load of the $m$th microgrid system.

In the above coalitional operation model, when all microgrids form as one grand coalition, DGs, energy storages and loads owned by different microgrid operators are dispatched indiscriminately in a centralized manner. In this way, neighboring microgrids can provide energy support to each other via power exchange, and share excessive resources like low-cost renewable energy, which consequently lead to reduction of total operation cost. On the other hand, the individual benefit of each microgrid operator is downplayed under the global optimization goal of minimizing the total cost. In order to guarantee the interest of local microgrid operators, a fair cost allocation method is proposed in this paper, in which case any microgrid operator, and any subset of microgrid operators can receive some cost saving by participating in the grand coalition. The detailed modeling of the cost allocation will be demonstrated in Section 4.

3. Linearized optimal power flow for distribution with multi-microgrid coalition

As has been observed in Section 2, Eqs. (9) and (10), the active and reactive network losses $P_{load}(t)$ and $Q_{load}(t)$ are nonlinear and non-convex terms, which adds to model complexity. To improve computation efficiency, we apply the linearization technique derived from [33] in the model, which are presented as follows:

For the $k$th line in the distribution system, the active and reactive line flow can be calculated as follows:

$$P_{k,t}(t) = \sum_{j=j+1}^{n} Sub(k+1, j+1) - (P_{t}(t) - P_{t}^{ij}(t)) + \sum_{j=k+2}^{n} Sub(k, j+1) - P_{t}^{ij}(t)$$

$$Q_{k,t}(t) = \sum_{j=j+1}^{n} Sub(k+1, j+1) - (Q_{t}(t) - Q_{t}^{ij}(t)) + \sum_{j=k+2}^{n} Sub(k, j+1) - Q_{t}^{ij}(t)$$

In Eqs. (15) and (16), both $j$ and $k$ are bus indices of the distribution system, $n$ is the set of buses. $Sub$ is a $n \times n$ matrix, where each element $(k+1, j)$ denotes if bus $j$ belongs to the sub-tree of bus $(k+1)$.

The active and reactive line loss in Eqs. (15) and (16) can be expressed as the following linearized function of bus generation:

$$P_{loss}^{ij}(t) \approx P_{loss}^{ij+1}(t) + \sum_{j=1}^{n} \frac{\partial P_{loss}^{ij-1}(t)}{\partial P_{ij}(t)} (P_{ij}(t) - P_{ij}^{0}(t)) + \sum_{j=1}^{n} \frac{\partial P_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)} (Q_{ij}(t) - Q_{ij}^{0}(t))$$

$$Q_{loss}^{ij}(t) \approx Q_{loss}^{ij+1}(t) + \sum_{j=1}^{n} \frac{\partial Q_{loss}^{ij-1}(t)}{\partial P_{ij}(t)} (P_{ij}(t) - P_{ij}^{0}(t)) + \sum_{j=1}^{n} \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)} (Q_{ij}(t) - Q_{ij}^{0}(t))$$

In Eqs. (17) and (18), the partial derivatives, i.e. $\frac{\partial P_{loss}^{ij-1}(t)}{\partial P_{ij}(t)}$, $\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$, $\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$, and $\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$, are called loss factors for distribution (LF-D), which describe the sensitivity of the $(j-1)^{th}$ line loss to the $i^{th}$ bus generation. The LF-D can be obtained as follows:

$$P_{loss}^{ij}(t) = \frac{P_{loss}^{ij+1}(t) + Q_{loss}^{ij+1}(t)}{\partial P_{ij}(t)} \frac{\partial Q_{loss}^{ij+1}(t)}{\partial Q_{ij}(t)}$$

The LF-D is calculated as the partial derivative of line losses to the bus generation:

$$\frac{\partial P_{loss}^{ij}(t)}{\partial P_{ij}(t)} = \left(2P_{ij}(t)\frac{\partial P_{ij}(t)}{\partial P_{ij}(t)} + 2Q_{ij}(t)\frac{\partial Q_{ij}(t)}{\partial P_{ij}(t)}\right) \frac{n}{V_{i}^{2}(t)} + \sum_{j=1}^{n} \frac{\partial P_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)} \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$$

$$\frac{\partial Q_{loss}^{ij}(t)}{\partial Q_{ij}(t)} = \left(2P_{ij}(t)\frac{\partial P_{ij}(t)}{\partial Q_{ij}(t)} + 2Q_{ij}(t)\frac{\partial Q_{ij}(t)}{\partial Q_{ij}(t)}\right) \frac{n}{V_{i}^{2}(t)} + \sum_{j=1}^{n} \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)} \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$$

In Eqs. (21)–(24), index $j$ is equal to $k + 1$, as shown in Fig. 2. LF-D is related to the generation shift factors (GSF), $\frac{\partial P_{loss}^{ij}(t)}{\partial P_{ij}(t)}$, $\frac{\partial Q_{loss}^{ij}(t)}{\partial Q_{ij}(t)}$, and $\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$, which is the sensitivity of line power flow to bus power injection, and can be calculated as follows:

$$\frac{\partial P_{loss}^{ij}(t)}{\partial P_{ij}(t)} = -Sub(k+1, j+1) + \sum_{j=k+2}^{n} Sub(k+1, j+1) - \frac{\partial P_{loss}^{ij-1}(t)}{\partial P_{ij}(t)}$$

$$\frac{\partial Q_{loss}^{ij}(t)}{\partial Q_{ij}(t)} = \left(\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}\right) \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$$

$$\frac{\partial Q_{loss}^{ij}(t)}{\partial Q_{ij}(t)} = \left(\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}\right) \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$$

$$\frac{\partial Q_{loss}^{ij}(t)}{\partial Q_{ij}(t)} = \left(\frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}\right) \frac{\partial Q_{loss}^{ij-1}(t)}{\partial Q_{ij}(t)}$$

It can be observed from Eqs. (21)–(28) that the calculation of generation shift factors and loss factors are nested within each other. Hence a recursive method is applied to obtain their values: to begin with, both GSF and LF-D are set to be zero. GSF are first calculated based on Eqs. (25)–(28). Then LF-D are calculated based on both GSF and the line power flow results from a linearized power flow model, which can be expressed as follows:

$$\begin{bmatrix} P^{ij} \\ Q^{ij} \end{bmatrix} = \begin{bmatrix} B_{ij} & B_{ij} \\ B_{ij} & B_{ij} \end{bmatrix} \begin{bmatrix} V^{ij} \\ V^{ij} \end{bmatrix}$$

In Eq. (29), $P^{ij}$ and $Q^{ij}$ are $(n-1) \times 1$ vectors accounting for active and reactive power injection for all buses except for the slack bus. $\Delta$ and $V$ are the voltage angle vector and voltage magnitude vector, both with dimension of $n \times 1$. The voltage angle and voltage magnitude at slack bus are set to 0 and 1 p.u., respectively. The $B$ matrix contains the resistance and reactance information of the system, and is expressed as follows:

$$B_{ij} = -\frac{r_{ij}}{r_{ij}} + \frac{x_{ij}}{r_{ij}^{2}} + \frac{x_{ij}^{2}}{r_{ij}^{2}} + \frac{x_{ij}}{r_{ij}^{2}}$$

$$B_{ij} = \sum_{j=1,j \neq i}^{n} \frac{r_{ij}}{r_{ij}} + \frac{x_{ij}}{r_{ij}^{2}} + \frac{x_{ij}^{2}}{r_{ij}^{2}} + \frac{x_{ij}}{r_{ij}^{2}}$$

Since the power injection equations do not include the slack bus, the $B_{ij}$ and $B_{ij}$ matrices are both $(n-1) \times n$ matrices with $i$ traversing from 2 to $n$ and $j$ traversing from 1 to $n$. Eq. (29) holds because originally we have:
The bus voltage angle difference along one branch is close to zero, and the bus voltage magnitude is around 1 p.u., Eqs. (32) and (33) can be simplified as follows:

\[
P_{ij} \approx \frac{-\eta_i x_{ij}}{r^2_{ij} + x^2_{ij}} \cos(\delta_i - \delta_j) + \frac{x^2_{ij}}{r^2_{ij} + x^2_{ij}} \frac{V_i}{x_{ij}}
\]

\[
Q_{ij} \approx \frac{-\eta_i x_{ij}}{r^2_{ij} + x^2_{ij}} \sin(\delta_i - \delta_j) + \frac{x^2_{ij}}{r^2_{ij} + x^2_{ij}} \frac{V_i}{x_{ij}}
\]

Which leads to the matrix formulation of the relationship between bus power injection and bus voltage, as is shown in Eq. (29). The power injection and bus voltage, as is shown in Eq. (29). The implementation of bus voltage constraints and the development of the associated algorithm is one of the main contributions of this paper. The implementation of bus voltage constraints and the development of the associated algorithm is one of the main contributions of this paper.

In this section we will present a Nucleolus-based core solution to the multi-microgrid cooperative operation case, as well as the associated calculation method.

In this paper, we use Nucleolus as the potentially feasible core solution to fair cost distribution among microgrids, as has been previously applied in [21, 34]. The Nucleolus allocation possesses several favorable properties: its existence is unique, and if the core set of the game is nonempty, it always belongs to the core. The main idea behind the Nucleolus allocation is to first find out the coalition that is most dissatisfied with the current allocation of the total cost, which implies that it may defect from the grand coalition. Then the grand coalition adjusts the allocations to minimize the dissatisfaction of this coalition. Nucleolus refers to this adjusted allocation. By conducting a Nucleolus allocation, the dissatisfaction of any coalition is minimized, and the stability of the grand coalition is ensured.

The dissatisfaction of a coalition under the current cost allocation is measured quantitatively as follows:

\[
g(c, x) = c^T x - v(c) = [x_1, x_2, ..., x_m, ..., x_N]^T, \text{ if } x = v(1) \tag{36}
\]

As we can see, in Eq. (36), the dissatisfaction is the difference between the total cost allocated to a coalition $c$ and the cost generated when $c$ operates separately from the grand coalition, where $x$ is the allocation vector indicating the cost distributed to each microgrid, and $v(1)$ is the total cost of the grand coalition, when every $c_m$ is equal to 1.

The expression is meaningful because a positive value of $g(c, x)$ indicates that staying in the grand coalition will cost the coalition $c$ more than the case in which it operates independently, therefore it may defect from the grand coalition. The adjustment model of the cost allocation based on dissatisfaction measurement can then be expressed as follows:
\[ f = \min_{x} \delta \]
\[ s.t. \quad \delta \geq g(x) = c^T x - v(x) \] 
\[ 0 < \Gamma c \leq M -1 = [c_1 c_2 \ldots c_{m-1} c_m], c_m \in [0,1] \]

In Eq. (37), the decision variables are \( c \) and \( x \). An auxiliary variable \( \delta \) is introduced to ensure that the maximum level of dissatisfaction among all coalitions is minimized, as shown in Eq. (38). In this way, the final allocation \( x \) satisfies the coalitional rationality. Eq. (39) encompasses all the coalitions except for the empty set and the grand coalition.

### 4.2. Finding Nucleolus solution via Benders decomposition algorithm

The difficult part of Eqs. (37)–(39) is that the number of coalitions grows exponentially with an increasing number of microgrid players, which makes it impossible to obtain all the values of \( v(c) \) in (36). To overcome this combinatorial explosion obstacle, we adopt a Benders Decomposition (BD) method, which has been previously applied in [35]. To begin with, Eq. (37) can be rewritten as follows:

\[ \min_{x} \{\max_{c} c^T x - v(c)\} \]  

(40)

Which is a min–max problem, and can be further decomposed into the following sub-problem and master problem:

**Sub-problem:**

\[ g^*(x) = \max_{c \in C \setminus \{\emptyset\}} \{c^T x - v(c)\} \]  

(41)

**Master-problem:**

\[ f^*(x) = \min_{x} \{\delta \} \]  

(42)

\[ s.t. \quad 0 < \Gamma c \leq M -1 \]  

(2)–(18)

\[ f^*(x) = \min_{x} \{\delta \} \]  

(43)

\[ s.t. \quad \delta \geq g^*(x(s)) + \delta^k g^*(x(s)) (x - x(s)) \quad \forall s = 1, 2, \ldots, k-1 \]  

(44)

\[ x_m = [0, 0, \ldots, 0] \]  

(45)

\[ \Gamma^T x = v(1) \]  

(46)

In the sub-problem (41), cost allocation \( x \) is fixed, and the variable is the binary coalition index \( c_m \). The objective is the difference between the allocated cost and the operating-alone cost, which stands for the dissatisfaction of coalition \( c \). Eq. (41) aims to find the coalition \( c \) that is most dissatisfied under the current cost allocation \( x \), and determines an upper bound of the optimum.

In the master-problem (43), the dissatisfaction \( g(x) \) from sub-problem (41) is used as input, and the variable is the cost allocation \( x \). The objective is to minimize the maximum dissatisfaction among all the previously found coalitions \( c \), and it determines a lower bound of the optimum. The algorithm iterates until the upper bound and the lower bound converge to the same value. Since the master problem is linear, the convergence of the algorithm is guaranteed. Eq. (44) is the Benders cut generated from the sub-problem in the form of first-order approximation of a Taylor series expansion, where \( \delta^k g^*(x(s)) \) is the partial derivative of \( g^* \) to \( x_m \) in the \( s^\text{th} \) iteration, and is equal to \( c_{i}^T \). Eqs. (45) and (46) are the representations of individual rationality and efficiency, respectively.

It can be observed that via Benders Decomposition method, there is no need to calculate \( v(c) \) for all coalitions \( c \), but only have to focus on the coalition that is most likely to refuse to cooperate, which considerably reduces computational efforts. Although the algorithm will take several iterations to converge, the number of iterations is still much less than the number of coalitions if applying an enumeration method. The computation efficiency of Benders Decomposition method will be further proved in the case study in Section 5.

It should be further pointed out that although the above Benders Decomposition method has already been applied in [35], the major difference between the current work and Ref. [35] is that an economic dispatch model is involved in this work, since each microgrid individual contains RESs, dispatchable distribute generators, and energy storage devices, among which the dispatch of the latter two are variables, which adds to higher complexity of the model, and leads to a multi-variable, high dimensional, and nonconvex problem. While Ref. [35] mainly focuses on renewable energy generation. In summary, in this paper we include a more complicated economic dispatch model within the fair allocation model to suit to multi-microgrid cooperation case, and efficiently combine optimal power flow linearization technique with Benders Decomposition to tackle the original high dimensional, nonconvex problem.

Another explanation that has to be made is that although there exists other core solutions in the cooperative game, such as Shapley value [36] and Nash-Harsanyi solution [37], these solutions have some undesirable properties like nonlinearity and nonconvexity, and cannot be efficiently decomposed as in the case of Nucleolus. Hence in this paper we mainly consider the Nucleolus allocation as the way of a fair cost distribution.

### 4.3. A panorama of solution process for fair cost allocation

It can be noticed that the above sub-problem (41) also includes the constraints from LOPF-D model, i.e. constraints (2)–(18), since the sub-problem involves economic dispatch of coalition \( c \) to find \( v(c) \), which requires an iterative algorithm to solve, as is stated in Section 3. The major difference between the model described by Eq. (41) and the multi-microgrid coalitional operation model (1) is that in the former only microgrid \( c \) is connected with the distribution system, while in the latter case all microgrids, namely the grand coalition is connected with the distribution system. This leads to the result that in Eq. (41), \( c \) is 0–1 binary variable, while in model (1) all \( c_m \) indices are 1, since all microgrids are included in the grand coalition. Another trivial difference is that Eq. (41) computes the difference between the cost allocated to the coalition \( c \) when it stays in the grand coalition, and the total operation cost of \( c \) when it operates alone, while Eq. (1) calculates the total operation cost of grand coalition.

The combination of LOPF-D constraints with Benders Decomposition method leads to a double-loop iteration algorithm to solve the fair cost allocation problem, as is shown in Fig. 4. As can be seen from the figure, the first iteration takes place in the sub-problem of fair cost allocation model, with a known cost allocation \( x \). The loss factors are constantly updated based on the current operating points, until the objective function \( g(x) = c^T x - v(c) \) reaches a fixed value. After the first iteration converges, we get the coalition \( c \) that has the highest dissatisfaction with the current cost allocation \( x \). We input \( g(x) \) into the master problem and readjust the cost allocation \( x \) to minimize this dissatisfaction. If the output from the master problem, which is the minimized dissatisfaction, does not equal the result from the sub-problem, we input the updated cost allocation \( x \) calculated from the master problem into the sub-problem, and repeat the above steps. The iterative calculation of the sub-problem and the master problem constitutes the second loop. If solving the multi-microgrid coalitional operation problem (1)–(18), only the first loop is required, since the problem only calculates the total operation cost and there is no decomposition of problem.

### 5. Case study

In this paper, the above established cooperative operation model of the multi-microgrid system and the associated cost allocation method are tested on both IEEE 33-bus distribution system and IEEE 123-bus distribution system. The first case is used to verify the economic
effectiveness of multi-microgrid cooperation as well as the stability of the formulated grand coalition. The second test case is intended to demonstrate the computation efficiency of Benders Decomposition method for cost allocation.

### 5.1. Simulation results of IEEE 33-bus distribution system

In this test case, ten microgrids are connected to the distribution system at different buses, and are formed as one grand coalition under ex-ante binding contracts, where there exist both electrical energy exchange and thermal energy change among coalition members. The multi-microgrid system topology has already been shown in Fig. 1. The power base of the system is 1 MVA. The voltage base is 12.66 kV. The parameters for DGs and energy storages are given in Table 1. The renewable energy generators, i.e. wind turbines and PVs, are assumed to work at MPPT mode with zero cost. Wind speed data and solar irradiation data are acquired from [38]. Load data is acquired from [39].

#### 5.1.1. Comparison between LOPF-D and ACOPF

As discussed in Section 3, the coalitional operation model of a multi-microgrid system is established as a LOPF-D problem. To ensure the accuracy of the proposed LOPF-D model, we compare the results of the coalitional operation model (1)–(18) with a conventional ACOPF model. We simulate the coalitional operation of the multi-microgrid system for 7 consecutive days, with the time interval set as 1 h, which is 168 h in total. To fully validate that the benefits from coalitional operation for each microgrid is not occasional, and can be maintained in the long term, we choose the time horizon as 168 h instead of the 24 h used in the daily schedule. The comparison of the results from LOPF-D and ACOPF are shown in Table 2.

In Table 2, the total cost refers to the operation cost for the simulated 168 h. The relative error of bus voltage, active and reactive network losses are the maximum relative error among all the buses and over all the time intervals. As seen from the table, the results from LOPF-D is very close to the one from ACOPF, which veriﬁes the accuracy of the former. Furthermore, it should be noted that the calculation time of LOPF-D model is 4 times faster than the ACOPF model, which substantiates its high computation efficiency.

#### 5.1.2. Optimal cost allocation among microgrids

We run the above multi-microgrid coalitional operation model and fair cost allocation model in two different scenarios: a winter scenario and a spring scenario. The two are different from each other in parameters including solar radiation, wind speed, electrical load and thermal load. The simulation length for both scenarios are 7 days, or 168 h. The aim is to verify the economy of multi-microgrid cooperation over a long-time scale in diversified real-world scenarios. The final cost savings for each microgrid member in the two scenarios are presented in Table 3:

### Table 1

<table>
<thead>
<tr>
<th>DG</th>
<th>(P_{\text{max}}^{\text{DG}}) (kW)</th>
<th>(P_{\text{max}}^{\text{DG}}) (kW)</th>
<th>(\eta) (%)</th>
<th>(\nu)</th>
<th>(C_{\text{fuel}}) ($/kWh)</th>
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<td>60</td>
<td>33</td>
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<tr>
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<td>100</td>
<td>85</td>
<td>–</td>
<td>–</td>
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<tr>
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<td>Capacity (kWh)</td>
<td>(P_{\text{max}}^{\text{Storage}}) (kW)</td>
<td>(\eta) (%)</td>
<td>Initial state (kWh)</td>
<td>(\Delta t) (h)</td>
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<tr>
<td>TES</td>
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#### Table 2

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<td>Active network losses: 0.4761</td>
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<td>Reactive network losses error: 0.4241</td>
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<td>Calculation time (s)</td>
<td>LOPF-D: 11.9291</td>
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<td>ACOPF: 57.7869</td>
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#### Table 3

<table>
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<th>No.</th>
<th>Nucleolus allocation ($)</th>
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<th>Cost saving (%)</th>
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<th>Independent operation ($)</th>
<th>Cost saving (%)</th>
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<tr>
<td></td>
<td>18</td>
<td>579.0589</td>
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<td>5</td>
<td>582.0589</td>
<td>497.0653</td>
<td>12.4137</td>
</tr>
</tbody>
</table>

| Spring |                          |                            |                 | 1   | 174.4132                 | 204.9736                   | 14.91            |
|        | 6                        | 108.6317                   |                 | 4   | 278.315                  | 278.6241                   | 0.11             |
|        | 10                       | 225.1895                   |                 | 5   | 203.6429                 | 229.6581                   | 11.33            |

Fig. 4. Flow chart of fair cost allocation process.

---

As is shown in the table, under the coalitional operation, each microgrid receives some cost saving in different scenarios, which substantiates the economic efficiency of multi-microgrid cooperation. Notice that in both winter and spring scenario cases, microgrid 2 has a negative cost under both Nucleolus allocation and independent operation. This is because when microgrid 2 has extra power supply, it can sell power back to the distribution system to support power consumption in its neighboring microgrids and make profits.

5.1.3. Economy analysis of multi-microgrid cooperation

We summarize two reasons behind the operation economy of the multi-microgrid coalitional operation. First of all, mutual power exchange among microgrids.

Fig. 5. Comparison of MT generation in coalitional operation and independent operation.

Fig. 6. DSO power exchange with transmission system.

Fig. 7. Daily dynamic changes of TES state of charge in coalitional operation case.
exchange among local microgrids increases the utilization efficiency of the zero-cost renewable energy and consequently reduces generation cost.

We first take a look at the micro turbine generation of microgrids in different situations. Fig. 5 demonstrates the micro turbine generation of microgrids in the winter scenario in both coalitional operation case and independent operation case, as well as the solar thermal generation. In the independent operation case, each microgrid has to supply the local thermal load on their own, with no power exchange with other microgrids. As can be observed from the three figures, during the time period with high solar thermal energy (i.e., 9 a.m.–4 p.m.), the coalitional operation case shows much less micro turbine generation than the independent case (the former is 0). This is because when forming as one grand coalition, thermal energy is transferred from microgrids with surplus solar panel generation to the ones with higher thermal load in the grand coalition, while in the independent case, since there is no local energy exchange, the more expensive micro turbine has to be applied to provide thermal energy.

The utilization of wind power in the multi-microgrid coalitional operation case is further demonstrated in Fig. 6. As can be observed from the figure, the power exchange between DSO and transmission system, $P_{grid}$, follows the tendency of the microgrid demand $P_{m}^{in}$. In addition, when the wind power generation is high, i.e. in the 17th, 84th, 85th and 98th hour, DSO purchased less power from the transmission system; while during other periods with lower wind power generation, more power is sent to the DSO to support the microgrid power demand. The correlation coefficients between DSO power exchange and microgrid demand is 0.6124, and the correlation coefficient between DSO power exchange and wind power is $-0.7253$, which validates the above observations. Hence it can be concluded that the interconnections among microgrids in the coalitional operation case makes it possible for wind power share, which in return reduces power purchase cost.

The second reason for the cost saving effects is related to the energy storage devices in the microgrids, and is shown in Figs. 7 and 8:

Fig. 7. Daily dynamic changes of TES state of charge in independent operation case.

Fig. 8. Daily dynamic changes of TES state of charge in coalitional operation case.

The above two figures demonstrate the daily dynamic changes of four thermal energy storages owned by MG1-MG4 in the winter scenario and in both coalitional operation case and independent operation case. As is shown in the figure, the energy storages reaches their maximum capacity more often in the coalitional operation case than in the independent operation case, which is especially evidently shown in TES4. This is because when operating cooperatively, the surplus thermal solar power of the microgrid without energy storage can be fully stored by the energy storage of another microgrid via local power exchange, therefore makes full use of zero-cost renewable energy and saves generation cost. While in the independent operation case, since each microgrid has a large amount of thermal load to supply, there is less extra thermal power to be stored.

Fig. 9. Bus voltage increase due to multi-microgrid penetration (winter scenario).
Further comments on the above analysis is that although in Fig. 1 the 10 microgrids are connected with the distribution system at specific locations, if randomly located, the economy of cooperative operation will still take place because the above two reasons behind the operation economy improvement does not relate to microgrid locations. This has been proved by simulations with microgrids arbitrarily located in different buses. For the sake of conciseness, the simulation results, which are much similar to Table 3 are skipped in the paper.

5.1.4. Impacts of multi-microgrid penetration on distribution system

We first investigate the impact of multi-microgrid penetration on the system losses. Network losses rate is implemented here, which is defined as the ratio of network work losses to the total power generation. In the winter scenario case, when there is no microgrids in the distribution system, the network losses rate is 4.65%; with 10 microgrids, the maximum hourly network losses rate over the entire simulation time span (168 h in total) is 4.39%. Hence it can be concluded that multi-microgrid penetration can help reduce network losses. This is because local energy consumption can be supplied by DGs in microgrids, therefore avoids long-distance power transmission and decreases network losses.

We further consider the impact of multi-microgrid penetration on system voltage level. Fig. 9(b) demonstrates the percentage of bus voltage increase during the entire simulation time span, with the maximum voltage increase percentage reaching 1.11%. Notice that at some buses, the voltage increase is below zero, which indicates a voltage decrease. This is because of the penetration of the microgrid demand. Still, Fig. 9(a) shows that all the bus voltage levels are within the feasible region [0.95pu, 1.05pu], which indicates that multi-microgrid penetration can provide reliable voltage support to the distribution system.

5.2. Simulation results of IEEE 123-bus distribution system

One of the key contributions of our paper lies in the application of Benders Decomposition method to overcome the combinatorial explosion in fair cost allocation among microgrid members. To fully verify the efficiency of the applied algorithm, we further test the coalitional operation model and the fair cost allocation model on a larger-scale 123-bus distribution system, with 30 microgrids involved in the cooperation, which leads to a tremendous number of coalitions, i.e. $2^{30} - 1$ in total. The topology of the 123-bus system is shown in Fig. 10:

In Fig. 10, 30 microgrids are connected to IEEE-123 bus distribution system and operate as one grand coalition. Similarly to the 33-bus test case, in this test case the multi-microgrid cooperation is simulated in two scenarios with a time span of 168 h. The comparison with ACOPF and final cost allocations are shown in Tables 4 and 5:

As can be seen from Table 4, the LOPF-D has adequate accuracy even on a larger system with far more variables, and the computation speed is more than 20 times faster than the ACOPF. In Table 5, in the spring scenario, MG9-10, MG19-20, MG29-30 receive zero cost saving. Although those microgrids cannot receive any cost savings in the spring scenario, they indeed benefit from coalitional operation in winter scenario (the cost saving percentage is greater than 0). If viewed from a long-term perspective, they would still be willing to stay in the grand coalition. For all the other microgrids, they receive some cost savings in both winter scenario and spring scenario by cooperation.

The computation efficiency of the Benders Decomposition method in finding Nucleolus cost allocation is demonstrated as follows: in the sub-problem (41), the number of constraints is 115,839 and the total number of variables is 113,431, including 30 binary variables; in the master-problem (43), the total number of variables is 30, the number of constraints grows with each iteration. The simulation is carried out on a hybrid platform, MATLAB 2016a plus GAMS 24.7, where MATLAB is
In the 123-bus test case, the total number of coalitions is $2^{30} = 1.0737 \times 10^9$. From Table 6, we can see that only a small number of coalitions needs to be checked before reaching the core solution, i.e. 278 in the winter scenario and 178 in the spring scenario. Therefore, we may safely conclude that the Benders Decomposition method holds considerable computation efficiency and can fairly be applied to large-scale system simulation.

6. Conclusion

This paper highlights the potential advantages of cooperation among multiple microgrids with different types of distributed energy resources at the distribution system level. The main contributions of this paper are summarized as follows:

(1). Inspired by cooperative game theory, a coalitional operation model of grid-connected microgrids is constructed to minimize the total operation cost. Simulation results verify that via local power exchange among microgrids, the utilization efficiency of renewable energy and also energy storage devices can be increased, which contribute to the generation cost reduction;

(2). To secure the stability of the multi-microgrid coalition, a fair allocation of the total operation cost among microgrids is further proposed, which is a core solution to the cooperative game. Benders decomposition algorithm is applied to obtain the core solution with high computational efficiency. In this way, the local optimum is achieved to guarantee the benefit of each microgrid player;

(3). We apply a linearized optimal power flow for distribution (LOPF-D) model in both multi-microgrid coalitional operation model and fair cost allocation model to include distribution network losses, since at distribution level the losses accounts for a significant part. Comparison with the ACOPF model verifies the accuracy of the proposed LOPF-D model as well as its computation efficiency.

The proposed work and conclusions hold considerable implications for real-world applications: (1) with the increasing penetration of the uncertain and intermittent renewable energy into the distribution system, a coalitional operation mode of multi-microgrid provides a viable solution to efficiently consume the surplus green power in case of over generation, which can reduce the fuel cost of dispatchable generators and also the dependence on the main grid for power supply; (2) the propose Nucleolus cost allocation method can realize local optimum and guard the interest of individual microgrid, which constitutes a strong prerequisite to fully realize a fair and profitable multi-microgrid cooperation. Hence, the proposed model and solution can be well explored for a practical application.

Acknowledgement

This work is partly supported in part by SGCC Science and Technology Program and in part by CURENT, a US NSF/DOE Engineering Research Center under the NSF award EEC-1041877.

Table 5

<table>
<thead>
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<th>No.</th>
<th>Nucleolus allocation ($)</th>
<th>Independent operation ($)</th>
<th>Cost saving (%)</th>
<th>No.</th>
<th>Nucleolus allocation ($)</th>
<th>Independent operation ($)</th>
<th>Cost saving (%)</th>
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<tbody>
<tr>
<td>Winter</td>
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<td></td>
<td></td>
<td>Spring</td>
<td></td>
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Table 6

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<tr>
<td>Spring</td>
<td>178</td>
<td>7482</td>
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used for creating input data profiles and recording computation results. The above min–max problem is solved by CPLEX on GAMS. The hardware environment is a laptop with Intel®Core™i5-6300U 2.4 GHz CPU, and 4.00 GB RAM. Computation time and iterations for calculating the Nucleolus cost allocation in each scenario is provided in Table 6:
References


[34] Dabbagh SR, Sheikh-El-Eslami MK. Risk-based profit allocation to DERs integrated with a virtual power plant using cooperative Game theory. Electric Power Syst Res 2015;121:368-78.


